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Kosterlitz–Thouless-type metal–insulator transition in two-dimensional layered media with long-range correlated disorder

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Abstract. We study the transport properties of two-dimensional striped media in the x–y plane with disorder that has long-range correlation along the y-direction. This long-range correlation is reflected by a special form of the Fourier transformation of the spatial distribution of the site energies $S(k) \propto 1/k^{\alpha}$ with k being the wave vector in the y-direction. Using the transfer-matrix method, we show that there is a continuous line of fixed points for $\alpha > 2$ indicating that the system undergoes a disorder-driven Kosterlitz–Thouless-type metal–insulator transition.

1. Introduction

The problem of Anderson localization in randomly layered media with isotropic randomness has been extensively studied in recent years [1, 2]. Physically, this system describes the Earth's subsurface and superlattices with lateral inhomogeneities. The study of localization phase diagrams in three dimensions (3D) has indicated that, when the ratio of the strength of the layer randomness to that of the isotropic randomness is larger than a certain critical value, the system behaves in a 1D-like fashion and all states are localized [1]. These results are in remarkable contrast to those for the anisotropic hopping model, where metal–insulator transitions (MIT) were found to occur at any anisotropy [3].

Recently, extensive attention has been attracted to the delocalization problem in low-dimensional systems with correlated disorder. The scaling theory of localization predicts that all states are localized in one or two dimensions for any amount of disorder [4]. However, a series of 1D correlated disordered systems have been studied which exhibit a set of delocalization states [5–8]. In the random-dimer model [5, 6] where the impurities are randomly embedded in nearest-neighbour pairs in an originally pure host chain, $\sqrt{N}$ of the electronic stationary states are extended over the entire lattice. Absence of localization has also been reported to occur in disordered chains with correlated off-diagonal interactions [7, 8]. More recently, de Moura and Lyra [9] studied the localization properties of the 1D Anderson model with long-range-correlated disorder. The on-site energy landscape is generated by considering the potential as the trace of a fractional Brownian motion with a specified spectral density $S(k) \propto 1/k^{\alpha}$. It has been found that the localization length diverges for $\alpha > 2$ within a finite range of energy values exhibiting an Anderson transition with mobility edges separating localized and extended states.

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The purpose of the present paper is to study how the localization behaviour in a 2D x–y plane is affected by the long-range correlation of disorder. This is of particular interest because 2D is the critical dimensionality in the scaling theory. We consider the correlation of random on-site energies in one direction for which the propagation of electrons is investigated. To do this we use a striped geometry with stripes along the x-direction, similar to the layered one in the 3D systems used in references [1, 2]. The random on-site energies are considered to be a superposition of the isotropic random site energies which have the same distribution in both directions as in the ordinary Anderson model and the random stripe energies which are arranged in such a sequence that describes the trace of a fractional Brownian motion with a specified spectral density $S(k_1) \propto 1/k_1^{\alpha}$ with k_1 being the wave vector in the y-direction normal to the stripes. Thus, within a stripe the randomness is uncorrelated, exhibiting a white-noise spectrum $S(k_2) \propto k_2^0$ with k_2 being the wave vector in the x-direction. There is a correlated component of the randomness in the y-direction from stripe to stripe. We calculate the localization length and the conductance of such systems with the transfer-matrix method. We find that all one-electron states remain localized for any amount of isotropic disorder for $\alpha < 2$, but there exists a line of critical points for $\alpha > 2$ indicating that the system undergoes a disorder-driven Kosterlitz–Thouless-type (KT) metal–insulator transition. By comparison with the results for the 1D system where a similar correlation of disorder exists [9], the energy range of the extended states is remarkably enlarged by increasing the dimensionality from 1D to 2D. On the other hand, for 2D systems the finite-size scaling *ansatz* can be more easily adopted and it shows the KT nature of the transition in the present structure of the disorder correlation.

This paper is organized as follows: in the next section we describe the Hamiltonian and basic formalism for the 2D disordered system with long-range correlation, in section 3 the numerical results are illustrated, and the last section is devoted to a brief summary and discussion.

2. Model of 2D long-range-correlated disorder

We consider a 2D tight-binding Anderson model on a square lattice

$$H = \sum_{i,m} \epsilon_{im} |im\rangle\langle im| + \sum_{\langle im,\, jn\rangle} t\,(|im\rangle\langle jn| + |jn\rangle\langle im|) \tag{1}$$

where i and m are integers numbering the position of a site in the x- and y-directions, respectively, $\langle im,\, jn\rangle$ denotes nearest neighbours on the lattice, t is the hopping matrix element, and ϵ_{im} the site energy at site (i, m). In what follows we set t to be the unit of energy. The stripes are along the x-direction, and we assume that there is a long-range correlation of the disorder in the y-direction from stripe to stripe. In such a structure the random on-site energy ϵ_{im} consists of two parts [1, 2]:

$$\epsilon_{im} = \eta_m + \nu_{im} \tag{2}$$

where ν_{im} is an isotropic random number that varies independently from site to site with a flat probability distribution within the range $[-W/2,\, W/2]$, and η_m describes the stripe energy which is constant for sites lying in the mth stripe but is randomly distributed from stripe to stripe in a long-range-correlated manner. In order to describe this long-range correlation we assume that the stripe energy η_m describes the trace of a fractional Brownian motion with a specified spectral density $S(k) \propto 1/k^{\alpha}$ for its spatial distribution. Thus η_m can be given by the relation [9]

$$\eta_m = \sum_{k=1}^{L/2} \left[k^{-\alpha} \left(\frac{2\pi}{L} \right)^{(1-\alpha)} \right]^{1/2} \cos\left(\frac{2\pi m k}{L} + \theta_k \right) \tag{3}$$

where L is the total number of stripes and θ_k are $L/2$ independent random phases uniformly distributed in the interval $[0, 2\pi]$. The exponent α is related to the Hurst exponent of the rescaled range analysis which describes the self-similar character of the series and the persistent character of its increments [10]. For $\alpha = 0$ one recovers 2D randomly striped media with uncorrelated disorder in the y-direction. In the case of $\alpha = 2$ the sequence of the stripe energies resembles the trace of the usual Brownian motion. In the absence of W, the momentum in the x-direction p_x is a good quantum number and for a given p_x the motion in the y-direction is a 1D one for which the extended states have already been confirmed within a finite range of energy for $\alpha > 2$ [9]. In the case of finite W, the system can be regarded as a 2D Anderson model superposed with a random stripe potential with long-range correlation. Our aim is to investigate the effect of such an additional correlated randomness on the states that are already localized in the original Anderson model.

3. Finite-size scaling analysis

In order to calculate the localization length of electrons, we use the finite-size-scaling method combined with the transfer-matrix technique [11]. We calculate the damping of wave functions in the y-direction (the direction with long-range correlation) for a long strip of size $M \times L$ where L is an extremely large length along the y-direction and M is the width of the strip. For a specific energy E, a $2M \times 2M$ transfer matrix T_m can easily be set up mapping the wave-function amplitudes at column $m - 1$ and m to those at column $m + 1$ in the strip. The propagation along the strip is therefore described by the product of the transfer matrices [11]

$$Q_L = \prod_{m=1}^{L} T_m. \tag{4}$$

The total transfer matrix of equation (4) has M pairs of eigenvalues whose absolute values of logarithms correspond to the Lyapunov exponents of the wave functions [11], $(\gamma_i, -\gamma_i)$ with $i = 1, 2, \ldots, M$. The largest localization length $\lambda_M(E)$ for energy E in a system with finite width M is given by the inverse of the smallest positive Lyapunov exponent. After obtaining all of the Lyapunov exponents for the strip of width M, the corresponding conductance in units of e^2/h along the y-direction of a square system with linear size M can be calculated as [11]

$$G = \sum_{i=1}^{M} \frac{2}{\cosh^2 \gamma_i L}. \tag{5}$$

In our numerical calculation, we choose the length of the strip L to be over 10^5 so that the self-averaging effect automatically takes care of the statistical fluctuations. We estimate and control these fluctuations from the deviations of the calculated eigenvalues of two adjacent iterations which show satisfactory suppression by increasing L. The finally obtained data have statistical errors less than the symbol size in the corresponding figures, so no explicit error bars are indicated.

We use the standard one-parameter finite-size scaling *ansatz* [11] to obtain the thermodynamic localization length ξ. According to the one-parameter scaling theory, the rescaled localization length λ_M/M can be expressed in terms of a universal function of M/ξ, i.e.,

$$\frac{\lambda_M}{M} = f\left(\frac{M}{\xi}\right). \tag{6}$$

We first study the localization behaviour of the system at the band centre $E = 0$ for typical values of α. In figure 1, we plot the rescaled localization length λ_M/M as a function of disorder W for $\alpha = 1.5$. The conductance G is calculated and shown in the inset of figure 1. Within

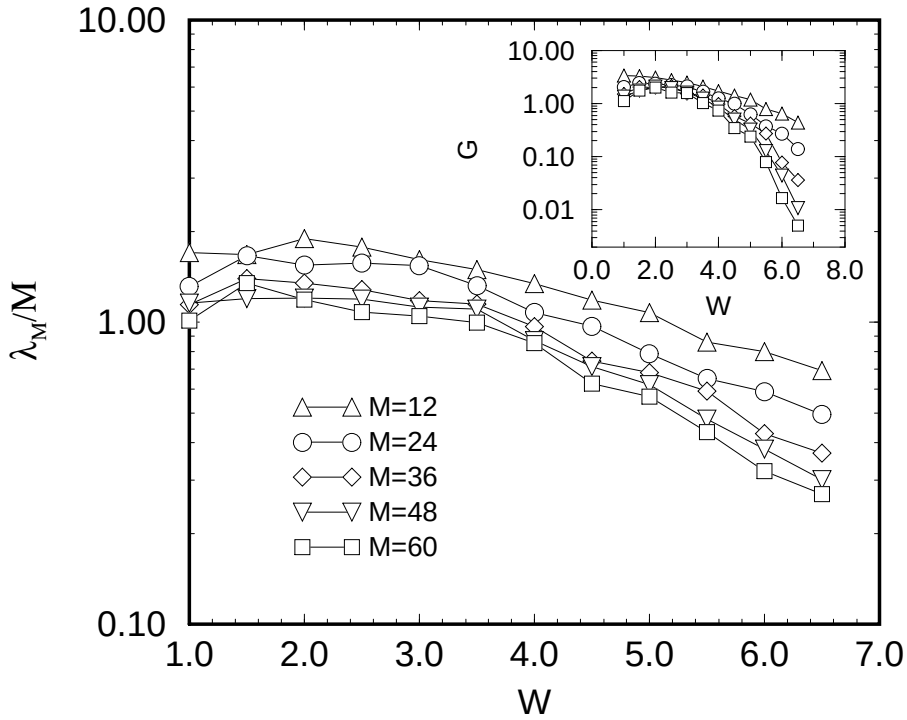

Figure 1. The rescaled localization length λ_M/M as a function of disorder W at $E = 0$ for $\alpha = 1.5$. Inset: the conductance G versus W.

numerical fluctuations, the curves of the smaller Ms are always above those of the larger Ms throughout the entire range of disorder W, indicating that there is no metal–insulator transition for any finite W. We also find that all of the states are localized at the band centre for $\alpha < 2$. However, this picture is qualitatively different for $\alpha > 2$. In figure 2 we show the curves for the same parameters as for figure 1 except that $\alpha = 2.5$. The striking difference from figure 1 is that all curves merge together for $W < W_c \simeq 3$. This shows that there is a line of critical points for $W < W_c$, indicating that the system undergoes a disorder-driven KT-type transition [12]. In the left-hand inset of figure 2, we extract the value of the localization length at the thermodynamical limit ξ from the scaling *ansatz* and successfully fit the data with

$$\xi \propto \exp(\theta_1/\sqrt{W - W_c})$$

indicating the exponential decay of the localization length on the insulating side by increasing the deviation from the transition point. This behaviour is a typical one for the KT transition [12, 13] and can provide further evidence for the KT-type transition. By fitting we find that $W_c = 2.8 \pm 0.2$, and $\theta_1 = 5.1 \pm 0.1$, the values of the parameters in the above exponential behaviour for the deviation of W from W_c on the insulator side.

In the usual continuous phase transition all of the curves in the finite-size-scaling studies should cross at a single point. One example of the delocalization–localization transition of this type in 2D is the case with spin–orbit interaction [14]. In the present case the transition is not a usual sharp continuous transition on the extended side. In this sense the phase on this side is not a true metallic state but only a critical one. The scaling behaviour can be investigated more generally by the calculation of the conductance of a square system. The β-function is defined as the logarithmic derivative of the conductance with respect to the system size. From

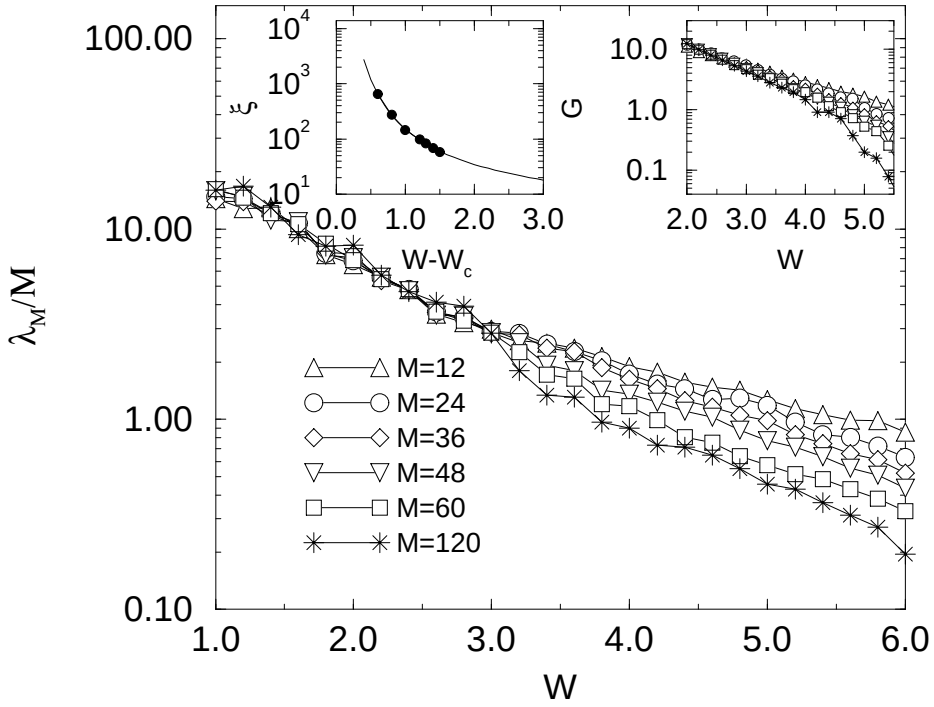

Figure 2. The rescaled localization length λ_M/M as a function of disorder W at $E = 0$ for $\alpha = 2.5$. Left-hand inset: the fit of the thermodynamic localization length ξ with the characteristic formula of the KT transition. Symbols show the numerical data. Right-hand inset: the conductance G as a function of disorder W.

equation (5) the β-function has the property $\beta \propto \mathrm{d}\ln(\lambda_M/M)/\mathrm{d}\ln M$ [11]. The curves for λ_M/M with different M for $W < W_c$ merging together imply that $\beta = 0$ in the metallic regime. Thus, it is expected that in this case the conductance G is also independent of the system size. In the right-hand inset of figure 2 we show the calculated conductance of a square system G as a function of W for different sizes. It is clear to see that the curves for G with different M merge together and are independent of M for $W < W_c$, as expected. In the present case the β-function may not be an analytical function of W in sweeping across W_c, owing to its zero value on the extended side and the exponential decay of the localization length on the insulator side. This is different from the ordinary continuous transition, and the β-function is used here only to provide a measure of the scaling behaviour of the system in the general scaling transformation.

Next we investigate the MIT in the band for a fixed disorder W. Presented in figure 3 is the rescaled localization length λ_M/M as a function of energy E with $W = 2$ for $\alpha = 2.5$. Figure 3 shows a behaviour of the λ_M/M–M dependence similar to that of figure 2—namely, all curves merge together for $E < E_c \simeq 1.3$. In the inset of figure 3, we fit ξ with the characteristic formula of KT transition

$$\xi \propto \exp(\theta_2/\sqrt{E - E_c})$$

with $\theta_2 = 3.2 \pm 0.1$ and $E_c = 1.4 \pm 0.12$.

We expect the mobility edge E_c to shift to higher energy with increasing value of α because the extended states are more favourable in this situation. This is exactly what we observe in our

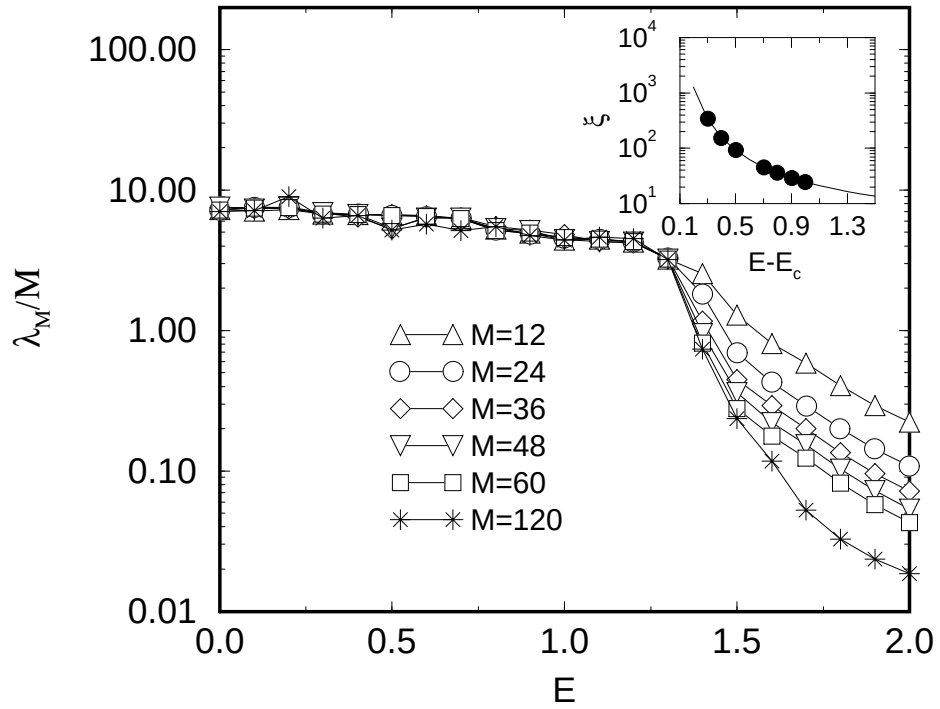

Figure 3. The rescaled localization length λ_M/M as a function of energy E with $W = 2$ for $\alpha = 2.5$. Inset: the fit of the thermodynamic localization length ξ with the characteristic formula of the KT transition.

calculation. In figure 4, we show the phase diagram in the (E_c, α) plane for a fixed disorder with $W = 2$. The states in region I are extended whereas the states in region II are localized. The range of energies corresponding to extended states increases with α, with the width of the extended phase saturating as $\alpha \to \infty$. By comparison with the results for a 1D disordered

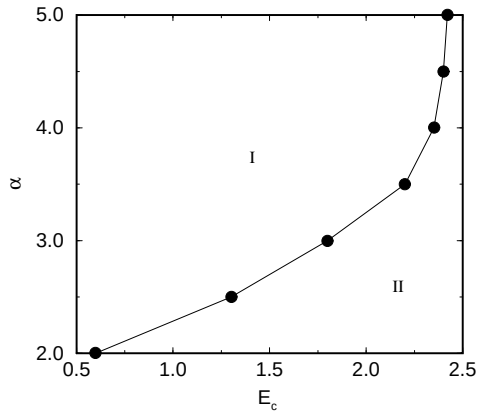

Figure 4. The phase diagram in the (E_c, α) plane for $W = 2$. The states in regimes I and II are extended and localized, respectively.

system with long-range correlation, it can be seen that the energy range for the extended states is remarkably enlarged. This is seen to be reasonable by considering that the 2D system is more favourable for the extended states than the 1D system. It is noteworthy that the correlation of disorder in one direction in 2D can even delocalize the states in the presence of an isotropic uncorrelated disorder which originally causes all the states to be localized.

In the present paper we only consider the propagation along the y-direction in which the long-range correlation of disorder exists. As for the propagation of electrons in the x-direction, the effect of the correlation can appear only if the width of the strip adopted in the finite-size-scaling study is comparable with the length of the correlation. This is difficult for the present case since the correlation range is quite large. The transition point may be different in different directions since the transition is no longer a usual second-order one and more than one transition point is allowed. It would be interesting to investigate this problem by using another method.

4. Conclusions

In summary, we have studied the localization properties in 2D randomly striped media with long-range-correlated disorder in one direction using the well-developed transfer-matrix method. We find that the system undergoes a disorder-driven KT-type metal–insulator transition once the stripe energy disorder distribution exhibits a power-law spectral density $S(k) \propto 1/k^{\alpha}$ with $\alpha > 2$. This result is in remarkable contrast to what one would expect for 2D disordered media with uncorrected disorder, where all states are localized for any amount of disorder [2].

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